1. $y = x^2 - k\sqrt{x}$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)
(Total 4 marks)

2. $f(x) = x^3 + 3x^2 + 5$.

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)dx$.

(4)
(Total 7 marks)

3. Given that $y = 2x^2 - \dfrac{6}{x^3}$, $x \neq 0$,

(a) find $\dfrac{dy}{dx}$,

(2)

(b) evaluate $\int_1^3 y\,dx$.

(4)
(Total 6 marks)

4. Given that $y = 6x - \dfrac{4}{x^2}$, $x \neq 0$,

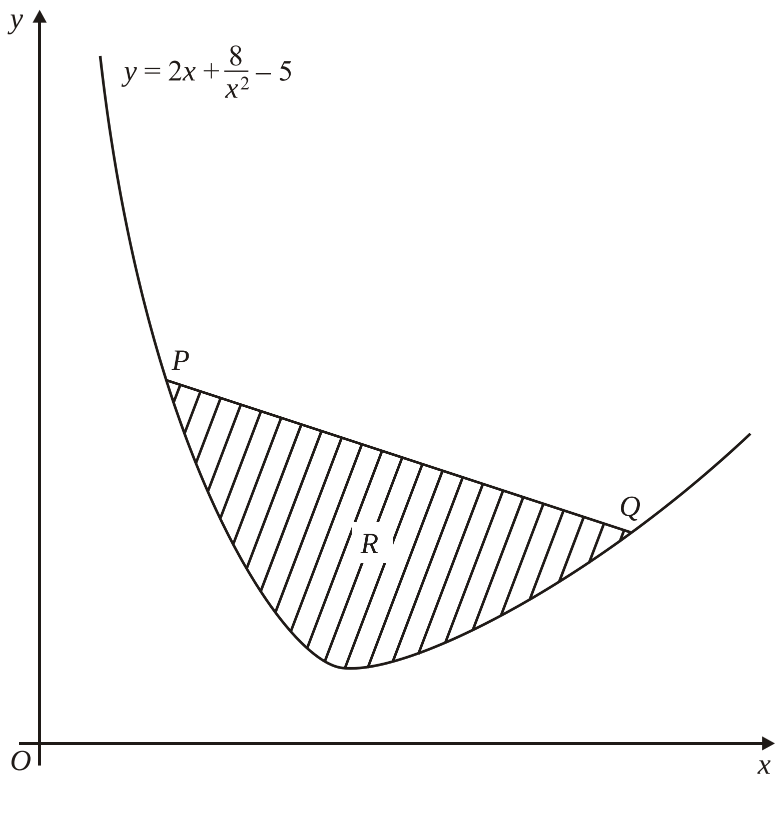
(a) find $\dfrac{dy}{dx}$,

(2)

(b) find $\int y \ dx$.

(3)
(Total 5 marks)

5.



This figure shows part of a curve C with equation $y = 2x + \dfrac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in the diagram, is bounded by C and the straight line joining P and Q.

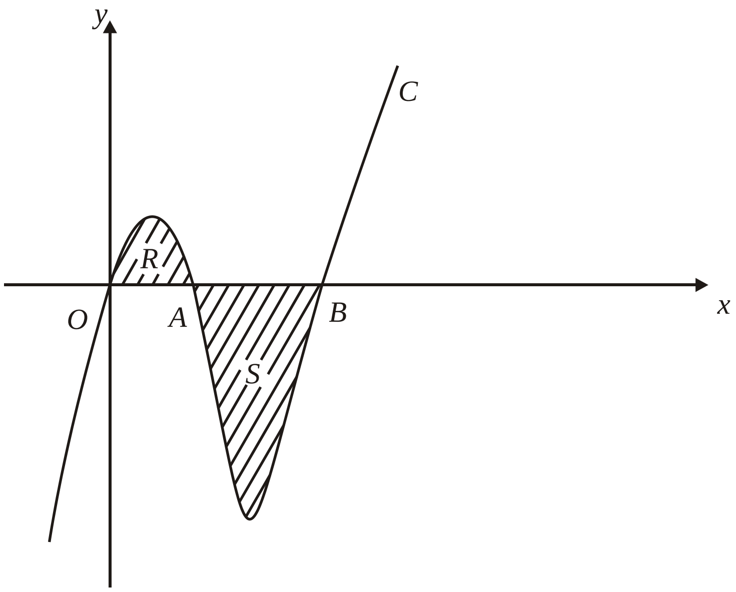
(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)
(Total 12 marks)

6.



The diagram above shows part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the x-axis at the origin O and at the points A and B.

(a) Factorise $\mathrm{f}(x)$ completely.

(3)

(b) Write down the x-coordinates of the points A and B.

(1)

(c) Find the gradient of C at A.

(3)

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

(d) Use integration to find the area of the combined regions R and S, shown shaded in the diagram above.

(7)
(Total 14 marks)

1. (a) $\left(\frac{dy}{dx}=\right) 2x-\frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. +*C*, is A0) M1 A1 2

Note

M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (c constant, $c \neq 0$)

(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero'

(The mark is allowed for : <, >, =, ≤, ≥) M1

$8-\frac{k}{4}<0$ $k > 32$ (or $32 < k$) Correct inequality needed A1 2

Note

Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes called y, and in this case the M mark can be given.

$\frac{dy}{dx} = 0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found.

Working must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.

[4]

2. (a) $f'(x) = 3x^2 + 6x$ B1

$f''(x) = 6x + 6$ M1, A1cao 3

Acceptable alternatives include

$3x^2 + 6x^1$; $3x^2 + 3\times2x$; $3x^2 + 6x + 0$

Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$)

$3x^2 + 6x + c$ or $3x^2 + 6x$ + constant (i.e. the written word constant) is B0 B1

M1 Attempt to differentiate their $f'(x)$; $x^n \to x^{n-1}$.

$x^n \to x^{n-1}$ seen in at least one of the terms. Coefficient of $x^{\cdots}$ ignored for the method mark.

$x^2 \to x^1$ and $x \to x^0$ are acceptable. M1

Acceptable alternatives include

$6x^1 + 6x^0$; $3 \times 2x + 3 \times 2$

$6x + 6 + c$ or $6x + 6$ + constant is A0 A1 cao

Examples

$f''(x) = 3x^2 + 6x$	B1 M0 A0	$f'(x) = x^2 + 3x$ $f''(x) = x + 3$	B0 M1 A0
$f'(x) = 3x^2 + 6x$ $f''(x) = 6x$	B1 M1 A0	$x^3 + 3x^2 + 5$ $= 3x^2 + 6x$ $= 6x + 6$	 B1 M1 A1
$y = x^3 + 3x^2 + 5$		$f'(x) = 3x^2 + 6x + 5$	B0
$\frac{dy}{dx} = 3x^2 + 3x$	B0	$f''(x) = 6x + 6$	M1 A1
$\frac{d^2y}{dx^2} = 6x + 3$	M1A0	$f'(x) = 3x^2 + 6x$ $f''(x) = 6x + 6 + c$	B1 M1 A0
$f'(x) = 3x^2 + 6x + c$ $f''(x) = 6x + 6$	B0 M1 A1		

(b) $\int (x^3 + 3x^2 + 5)dx = \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ M1, A1

$\left[\frac{x^4}{4} + x^3 + 5x\right]_1^2 = 4 + 8 + 10 - (\frac{1}{4} + 1 + 5)$ M1

$= 15\frac{3}{4}$ o.e. A1 4

Attempt to integrate f(x); $x^n \to x^{n+1}$
Ignore incorrect notation (e.g. inclusion of integral sign) M1

o.e.
Acceptable alternatives include
$\frac{x^4}{4} + x^3 + 5x; \frac{x^4}{4} + \frac{3x^3}{3} + 5x^1; \frac{x^4}{4} + \frac{3x^3}{3} + 5x + c; \int \frac{x^4}{4} + \frac{3x^3}{3} + 5x$

N.B. If the candidate has written the integral (either $\frac{x^4}{4} + \frac{3x^3}{3} + 5x$
or what they think is the integral) in part (a), it may not be rewritten in (b), but the marks may be awarded if the integral is used in (b). A1

Substituting 2 and 1 into any function other than $x^3 + 3x^2 + 5$ and subtracting either way round.
So using their $f'(x)$ or $f''(x)$ or $\int$ their $f'(x)dx$ or $\int$their $f''(x)$ dx will gain the M mark (because none of these will give $x^3 + 3x^2 + 5$).
Must substitute for all x s but could make a slip.

$4 + 8 + 10 - \frac{1}{4} + 1 + 5$ (for example) is acceptable for evidence of subtraction ('invisible' brackets). M1

o.e. (e.g. $15\frac{3}{4}$, 15.75, $\frac{63}{4}$)

Must be a single number (so $22 - 6\frac{1}{4}$ is A0). A1

Answer only is M0A0M0A0

Examples

$\frac{x^4}{4} + x^3 + 5x + c$	M1A1	$\frac{x^4}{4} + x^3 + 5x + c$	M1 A1
$4 + 8 + 10 + c - (\frac{1}{4} + 1 + 5 + c)$	M1	$x = 2,\ 22 + c$	
$= 15\frac{3}{4}$	A1	$x = 1,\ 6\frac{1}{4} + c$ (no subtraction)	M0 A0

$\int_1^2 \mathrm{f}(x)\mathrm{d}x = 2^3 + 3\times 2^2 + 5 - (1 + 3 + 5)$ M0A0, M0

$= 25 - 9$

$= 16$ A0

(Substituting 2 and 1 into $x^3 + 3x^2 + 5$, so 2nd M0)

$\int_1^2 (6x + 6)\mathrm{d}x = \left[3x^2 + 6x\right]_1^2$	M0 A0	$\int_1^2 (3x^2 + 6x)\mathrm{d}x = \left[x^3 + 3x^2\right]_1^2$	M0 A0
$= 12 + 12 - (3 + 6)$	M1 A0	$= 8 + 12 - (1 + 3)$	M1 A0

$\frac{x^4}{4} + x^3 + 5x$ M1 A1

$\frac{2^4}{4} + 2^3 + 5\times 2 - \frac{1^4}{4} + 1^3 + 5$ M1

(one negative sign is sufficient for evidence of subtraction)

$= 22 - 6\frac{1}{4} = 15\frac{3}{4}$ A1

(allow 'recovery', implying student was using 'invisible brackets')

(a) $\mathrm{f}(x) = x^3 + 3x^2 + 5$

$\mathrm{f}''(x) = \frac{x^4}{4} + x^3 + 5x$ B1M0A0

(b) $\frac{2^4}{4} + 2^3 + 5\times 2 - \frac{1^4}{4} - 1^3 - 5$ M1A1M1

$= 15\frac{3}{4}$ A1

The candidate has written the integral in part (a). It is not rewritten in (b), but the marks may be awarded as the integral is used in (b).

[7]

3.	(a)	$\frac{dy}{dx} = 4x + 18x^{-4}$	$x^n \mapsto x^{n-1}$	M1 A1	2
	(b)	$\int \left(2x^2 - 6x^{-3}\right)dx = \frac{2}{3}x^3 + 3x^{-2}$	$x^n \mapsto x^{n+1}$	M1 A1	
		$[...]_1^3 = \frac{2}{3} \times 3^3 + \frac{3}{9} - \left(\frac{2}{3} + 3\right)$		M1	
		$= 14\frac{2}{3}$ $\qquad \frac{44}{3}, \ \frac{132}{9}$ or equivalent		A1	4

[6]

4.	(a)	$\frac{dy}{dx} = 6 + 8x^{-3}$	M1 A1	2

M1 is for $x^n \rightarrow x^{n-1}$ in at least one term, 6 or x^{-3} is sufficient.
A1 is fully correct answer.
Ignore subsequent working.

	(b)	$\int y\,dx = \frac{6x^2}{2} + 4x^{-1} + C$	M1 A1 A1	3

M1: Correct power of x in at least one term (C sufficient)

First A1: $\frac{6x^2}{2}$ + C

Second A1: + $4x^{-1}$

[5]

5.	(a)	$\int (2x + 8x^{-2} - 5)dx = x^2 + \frac{8x^{-1}}{-1} - 5x$	M1 A1 A1	
		$\left[x^2 + \frac{8x^{-1}}{-1} - 5x\right]_1^4 = (16 - 2 - 20) - (1 - 8 - 5) \quad (= 6)$	M1	
		$x = 1$: $y = 5$ and $x = 4$: $y = 3.5$	B1	
		Area of trapezium = $\frac{1}{2}(5 + 3.5)(4 - 1) \quad (= 12.75)$	M1	
		Shaded area = 12.75 – 6 = 6.75	M1 A1	8

(M: Subtract either was round)

Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0.

Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.

Alternative:

$x = 1$: $y = 5$ and $x = 4$: $y = 3.5$ — B1

Equation of line: $y - 5 = -\frac{1}{2}(x - 1)$ $\quad y = \frac{11}{2} - \frac{1}{2}x$, subsequently used in integration with limits. — 3rd M1

$\left(\frac{11}{2} - \frac{1}{2}x\right) - \left(2x + \frac{8}{x^2} - 5\right)$ — 4th M1

(M: Subtract either way round)

$\int\left(\frac{21}{2} - \frac{5x}{2} - 8x^{-2}\right)dx = \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}$ — 1sr M1 A1ft A1ft

(Penalise integration mistakes, not algebra for the ft marks)

$\left[\frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}\right]_1^4 = (42 - 20 + 2) - \left(\frac{21}{2} - \frac{5}{4} + 8\right)$ — 2nd M1

(M: Right way round)

Shaded area = 6.75 — A1

(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)

(b) $\frac{dy}{dx} = 2 - 16x^{-3}$ — M1 A1

(Increasing where) $\frac{dy}{dx} > 0$; For $x > 2$, $\frac{16}{x^3} < 2$, $\therefore$ $\frac{dy}{dx} > 0$ — dM1; A1 — 4

(allow ≥)

<u>*Alternative for the last 2 marks in (b)*</u>:

M1: Show that $x = 2$ is a minimum, using, e.g., 2nd derivative.
A1: Conclusion showing understanding of " increasing", with accurate working.

[12]

6. (a) Correct method for one of the 3 factors. — M1

$x(x - 1)(x - 5)$ — M1 A1 — 3

Allow $(x \pm 0)$ instead of x.
(2nd M1 for attempting full factorisation)

(b) 1 and 5 — B1 ft — 1

(c) $\frac{dy}{dx} = 3x^2 - 12x + 5$ M1 A1

At $x = 1$. $\frac{dy}{dx} = 3 - 12 + 5 = -4$ A1 3

(d) $\int (x^3 - 6x^2 + 5x)dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$ M1 A1

Evaluating at one of their x value: $\frac{1}{4} - 2 + \frac{5}{2}\left(= \frac{3}{4}\right)$ M1 A1 ft

Evaluating at the other x value: $\frac{625}{4} - 250 + \frac{125}{2}\left(= -31\frac{1}{4}\right)$ A1

$[\ldots..]_5 - [\ldots..]_1$ or $[\ldots..]_1 - [\ldots..]_5$ M1

$$-31\frac{1}{4} - \frac{3}{4} = -32$$

Total Area $= 32 + \frac{3}{4} = 32\frac{3}{4}$ A1 7

If integrating the wrong expression in (d), (e.g. $x^2 - 6x + 5$), do not allow the first M mark, but then follow scheme.

[14]

1. The differentiation in part (a) of this question was usually completed correctly, although the $k\sqrt{x}$ term sometimes caused problems, with k being omitted or $\sqrt{x}$ misinterpreted.

It was clear in part (b), however, that many candidates did not know the condition for a function to be decreasing. Some substituted $x = 4$ into y rather than $\frac{dy}{dx}$ and some used the second derivative. Even those who correctly used $\frac{dy}{dx}$ were usually unable to proceed to a correct solution, either making numerical mistakes (often being unable to find the correct value of $4^{-\frac{1}{2}}$) or failing to deal correctly with the required inequality. The answer $k = 32$ was commonly seen instead of $k > 32$.

2. Part (a) was answered well with many correct solutions. A few candidates integrated f(x). Some candidates had difficulty differentiating the constant term. The most common incorrect solution was $f''(x) = 6x$. In part (b) a few candidates used f(x) or f′(x) as their integral. However, most integrated successfully and substituted accurately. Occasional arithmetic slips were seen.

3. The general standard of calculus displayed throughout the paper was excellent and full marks were common on this question. A few candidates took the negative index in the wrong direction, differentiating x^{-3} to obtain $-2x^{-2}$ and integrating x^{-3} to obtain $-\frac{x^{-4}}{4}$.

4. **Pure Mathematics P1**

The vast majority of candidates gained the method marks and many went on to score four or five marks; the four mark score was usually due to the omission of the arbitary constant in part (b).

Differentiation and integration of $-\frac{4}{x^2}$ caused problems for some candidates, and those who wrote y as $\frac{6x^3-4}{x^2}$ before differentiating were usually defeated.

Providing $8x^{-3}$ in part (a) and $+ 4x^{-1}$ in part (b) had been seen, subsequent errors such as writing these as $\frac{1}{8x^3}$ or $\frac{1}{4x}$ respectively, were not penalised, but it should be noted that such errors with indices were quite common.

Core Mathematics C1

This was another straightforward question for many candidates and the principles of differentiation and integration were understood by most. The negative power caused problems for some again, they realized that the term $-\frac{4}{x^2}$ needed to be written as $-4x^{-2}$ but did not always apply the rules successfully. In part (b) some could not simplify $-\frac{4x^{-1}}{-1}$ to $+4x^{-1}$ and a sizeable minority lost a mark for failing to include a constant of integration.

5. There were many very good solutions to part (a), scoring full marks or perhaps losing just one or two marks for slips in accuracy. It was disappointing, however, that so many candidates used an unnecessarily long method, finding the equation of the straight line through *P* and *Q*, then integrating, when it was much simpler to use the formula for the area of a trapezium (or equivalent). Integration techniques were usually sound, although $8x^{-2}$ was occasionally integrated to give $\frac{8x^{-3}}{-3}$. Sometimes candidates simply found the area under the curve and did not proceed any further.

Part (b) required candidates to use calculus to show that *y* was increasing for $x > 2$, and responses here often suggested a lack of understanding of the concept of an increasing function. Weaker candidates often made no attempt at this part, or simply calculated values of *y* for specific values of *x*. Usually, however, $\frac{dy}{dx}$ was found and there was some indication that a positive gradient implied an increasing function, but again a common approach was to find values of the gradient for specific values of *x*. Accurate and confident use of inequalities was rarely seen. Although there were some good solutions based on proving that $x = 2$ was a minimum, completely convincing and conclusive arguments in part (b) were rare.

6. High marks were often scored in this question. The factorisation in part (a) proved surprisingly difficult for some candidates, especially those who failed to use x as a factor. Some used the factor theorem to show that $(x - 1)$ was a factor, and then used long division, but failed to factorise $(x^2 - 5x)$. Even these, however, usually realised that 1 and 5 were the required x-coordinates in part (b). While most candidates found the gradient correctly in part (c), it was notable that others failed to realise that differentiation was needed.

Apart from arithmetic slips, the majority of candidates were able to integrate and substitute limits correctly in part (d), where the only real problem was in dealing with the negative value (region below the x-axis) for the integral from 1 to 5. Here, some tried to compensate for the negative value in unusual ways and never managed to reach an appropriate answer for the combined area.